LITERATURE CITED

1. N. S. Gryaznov, Principles of Coking Theory [in Russian], Metallurgiya, Moscow (1976).
2. Yu. V. Polezhaev and F. B. Yurevich, Thermal Protection [in Russian], Énergiya, Moscow (1976).
3. S. Madorskii, Thermal Degradation of Organic Polymers [Russian translation], Mir, Moscow (1967).
4. A. G. Shashkov, V. I. Tyukaev, and A. T. Nikitin, "Thermal diffusivity of composite materials at high temperatures," Int. J. Heat Mass Transfer, 14, No. 10, 1567-1574 (1971).
5. L. N. Aksenov, A. P. Bronshtein, G. N. Makarov, and O. F. Shlenskii, Summary of Documents of an All-Republic Scientific–Technical Conference: Problems of Processing Mineral Fuels, Minsk (1980), p. 85.
6. T. V. Komarova, S. D. Fedoseev, O. M. Petrova, and A. N. Seleznev, "Kinetics of pyrolysis of phenol-formaldehyde polymers," Prikl. Khim., No. 7, 1666-1669 (1977).
7. O. F. Shlenskii, Thermal Properties of Glass-Plastics [in Russian], Khimiya, Moscow (1973).
8. Shlenskii et al., "Features of the behavior of high-molecular-weight compounds at high heating rates," Dokl. Akad. Nauk SSSR, 253, No. 4, 921-925 (1980).
9. O. F. Shlenskii and G. E. Vishnevskii, Summary of Documents of an All-Union Scientific–Technical Conference: Heat and Mass Transfer in Power Plants, GKNT, Tula (1975), p. 87.
10. V. R. Regel', A. I. Slutsker, and V. A. Tomashevskii, Kinetic Theory of the Strength of Solids [in Russian], Nauka, Moscow (1973).
11. V. R. Regel' and O. F. Pozdnyakov, "Study of processes of the thermal and mechanical degradation of polymers," Mekh. Polim., No. 1, 16-32 (1975).
12. O. F. Shlenskii and G. E. Vishnevskii, "Thermogravimetric studies of the degradation of certain polymers by the impulse method," Dokl. Akad. Nauk SSSR, 246, No. 1, 151-153 (1979).
13. O. F. Shlenskii and G. M. Berman, "Study of the durability of polymeric materials," Energ. Stroit., No. 4, 49 (1979).

MOLECULAR TRANSFER

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Universal relations pertaining to molecular transfer of momentum, heat, and mass are derived on the basis of a special mathematical transformation.

Molecular transfer occurs widely in nature as well as in technology and, therefore, knowing the laws which govern it is particularly important. The laws governing the molecular transfer of momentum, heat, and mass cannot be derived through solution of known differential equations, because the system of these equations is generally not a closed one. Relations based on semiempirical theories or on processing of experimental data, on the other hand, contain empirical constants and are not general.

In [1, 2] universal relations for molecular transfer of momentum and heat have been derived with the aid of a special mathematical model. In this study those results will be refined and extended.

The region of molecular transfer will be defined as

$$x_2 \geqslant x \geqslant x_1 \geqslant 0, \quad \delta(x) \geqslant y \geqslant \delta_0(x) > 0, \tag{1}$$

where x is the longitudinal coordinate measurable on the solid surface (wall); y, transverse coordinate measured from the wall; $\delta(x)$, upper limit of this region (e.g., thickness of the turbulent boundary layer); and $\delta_0(x)$, lower limit of this region (thickness of the laminar sublayer).

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One can obtain universal relations for molecular transfer by transforming (normalizing) the function of two variables defined in region (1) into a generalized single-valued function U of one generalized variable R(x, y).

Since the thus obtained function U must remain a generalized one of parameters of the transferred substance and R must remain a generalized coordinate, hence

$$U = U(u,\ u_0,\ u_\delta), \tag{2}$$

$$R = R(y,\ \delta_0,\ \delta), \tag{3}$$

where $u_0 = u(x, \delta_0(x))$ and $u_\delta = u(x, \delta(x))$.

It follows from expressions (2) and (3) that the normalization can be reduced to certain mathematical operations on two groups of functions: u(x, y), $u_0(x)$, $u_\delta(x)$ and y, $\delta_0(x)$, $\delta(x)$.

We will introduce operator P performing on function u defined in region (1) the transformation

$$Pu = \frac{u - u_0}{u_\delta - u_0}\ . \tag{4}$$

In the U(R) representation two values of R must correspond to curves δ, δ_0 and two values of U must correspond to curves u_δ, u_0. According to expression (4), operator P meets this requirement.

As function U one can select a simple combination of u, u_0, and u_δ satisfying the relation (4), inasmuch as not the form of expression (2) by the relation U(R) is sought. One can, therefore, let

$$U = Pu. \tag{5}$$

According to expression (5), δ_0 is the scale of the y coordinate.

In the selection of δ_0 as scale for y it is possible to perform on the group of functions y, $\delta_0(x)$, $\delta(x)$ the mathematical operations

$$y \to y/\delta_0 \to f(y/\delta_0) \to Pf(y/\delta_0), \tag{6}$$

$$y \to f(y) \to Pf(y), \tag{7}$$

where f is an arbitrary function.

We do not consider here transformations of the $y \to (y - \delta_0) \to f(y - \delta_0) \to Pf(y - \delta_0)$ kind, inasmuch as this would exclude from the analysis functions f(z) not defined at z = 0.

The sought generalized relation U(R) must not depend on the possible route by which it is established, according to (6) or (7), and must thus be invariant with respect to transformations. Therefore,

$$Pf(y/\delta_0) = Pf(y). \tag{8}$$

From here it is easily established that function f(z) can only be of one of the two kinds

$$f_1(z) = \mathrm{const}\, z^n + \mathrm{const}, \tag{9}$$

$$f_2(z) = \mathrm{const} \ln z^n + \mathrm{const}, \tag{10}$$

with the argument z equal to either y or y/δ and $n > 0$.

Relations (10) and (4) yield

$$R = Pf_2(z) = \frac{\ln(y/\delta_0)}{\ln(\delta/\delta_0)}\ . \tag{11}$$

Relations (9) and (4) yield

$$R = Pf_1(z) = \frac{y^n - \delta_\delta^n}{\delta^n - \delta_0^n}\ . \tag{12}$$

Expression (11) differs from expression (12) in that it does not contain the parameter n. Therefore, one must select expression (11) for R and expression (5) for U in the generalized relation U(R).

It has been demonstrated in [1, 2] that

$$\frac{dU}{dR}=1, \tag{13}$$

$$U=R, \tag{14}$$

$$\frac{d\Theta}{dR}=1, \tag{15}$$

$$\Theta=R, \tag{16}$$

where $U = (u - u_0)/(u_\delta - u_0)$ and $\Theta = (T - T_0)/(T_\sigma - T_0)$.

Expressions (13) and (15) represent generalized relations describing molecular transfer of momentum and heat.

Analogously one obtains the generalized relation for molecular transfer of mass

$$\frac{dC}{dR}=1, \tag{17}$$

and its integral

$$C=R. \tag{18}$$

Here $C = (c - c_0)/(c_\delta - c_0)$ and c is the mean concentration of the substance.

The results ought not to be different when the operator

$$P'u=\frac{u-u_\delta}{u_0-u_\delta}=U' \tag{19}$$

is introduced and operations (6), (7) in selection of δ as scale for y are replaced with transformations

$$y\to y/\delta\to f(y/\delta)\to P'f(y/\delta), \tag{20}$$

$$y\to f(y)\to P'f(y). \tag{21}$$

As a result then we have

$$R'=\frac{\ln(y/\delta)}{\ln(\delta_0/\delta)}, \tag{22}$$

$$U'=R'. \tag{23}$$

From relation (14) we obtain

$$u^+=\frac{1}{\Psi_\delta}\ln(y^+/\delta_0^+)+u_0^+, \tag{24}$$

and from relation (23) we obtain

$$u^+=\frac{1}{\Psi_\delta}\ln(y^+/\delta^+)+u_\delta^+ . \tag{25}$$

Relations (24) and (25) yield

$$\frac{\ln(y^+/\delta_0^+)}{u^+-u_0^+}=\frac{\ln(y^+/\delta^+)}{u^+-u_\delta^+} . \tag{26}$$

Here $\Psi_\delta = \ln(\delta^+/\delta_0^+)/(u_\delta^+ - u_0^+)$. Consequently, the function

$$\Psi=\frac{\ln(y^+/\delta_0^+)}{u^+-u_0^+} \tag{27}$$

is invariant with respect to simultaneous replacement of u_0, δ_0 with u_δ, δ and thus does not depend on the choice of scales.

It is convenient to replace expression (14) with

$$\Psi=\Psi_\delta, \tag{28}$$

since $\Psi = \Psi(x, y)$ and $\Psi_\delta = \Psi_\delta(x)$.

From expression (28) we find that when

$$\tilde{\Psi}_\delta = \tilde{\Psi}_0 \tag{29}$$

there holds true the equality

$$\Psi_\delta = \tilde{\Psi}_\delta, \tag{30}$$

where $\tilde{\Psi}_\delta = \ln \delta^+/(u_\delta^+ - 1)$, and $\tilde{\Psi}_\theta = \ln \delta_o^+/(u_o^+ - 1)$.

It has been demonstrated in the first study [1] that at a zero pressure gradient in the turbulent core we have

$$\tilde{\Psi} = \tilde{\Psi}_\delta \tag{31}$$

or

$$\tilde{U} = \tilde{R}, \tag{32}$$

where $\tilde{\Psi} = \ln y^+/(u^+ - 1)$; $\tilde{U} = (u^+ - 1)/(u_\delta^+ - 1)$, and $\tilde{R} = \ln y^+/\ln \delta^+$.

On the diagram in Fig. 1 expression (32) is compared with experimental data from another study [3] pertaining to flow of mercury through a pipe in a longitudinal magnetic field at various values of the Hartmann number $N_{Ha} = Bd\sqrt{\sigma/\rho\nu}$. Here B is the magnetic induction; σ, electrical conductivity; and d, tube diameter. The experimental data appear to agree satisfactorily with expression (32).

The graph in Fig. 1 indicates that calculation of such a flow on the basis of the proposed mathematical model [1, 2] is rather simple, inasmuch as for determining the distribution of mean velocity and the shearing stress at the wall one must know only the relation $\tilde{\Psi}_\delta = \tilde{\Psi}_\delta(N_{Ha}^2/N_{Re_\delta})$ (where $N_{Re_\delta} = u_\delta\delta/\nu$) in addition to ν, ρ, δ, and u_δ.

On the diagram in Fig. 2 expression (32) is compared with experimental data of still another study [4] pertaining to turbulent flow through the initial segment of a rough pipe at various values of the ratio x/d, where x is the distance from the tube entrance. The experimental data here appear to be correlating satisfactorily with the theoretical relation (32).

The calculation of such a flow is based on known ν, ρ, δ, u_δ, and relation $\tilde{\Psi}_\delta = \tilde{\Psi}_\delta(x/d)$.

Accordingly, $\tilde{\Psi}_\delta$ and Ψ_δ are critical functions for a turbulent boundary layer.

The graph in Fig. 3 depicts the relation $\tilde{\Psi}_\delta = \tilde{\Psi}_\delta(\delta^+)$ according to data in study [5] on turbulent flow in a smooth pipe. Evidently $\tilde{\psi}_\delta \to \text{const} = 0.34$ as $\delta^+ \to \infty$. With sufficient accuracy, moreover, one can assume that at $\delta^+ \geq 10^3$ will be

$$\tilde{\Psi}_\delta = \frac{\ln \delta^+}{u_\delta^+ - 1} = 1/3. \tag{33}$$

According to the Blasius method for a laminar boundary layer [6], one obtains

$$\delta^+/u_\delta^+ = 1.60. \tag{34}$$

The graph in Fig. 3 indicates that function $\tilde{\Psi}_\delta$ calculated for a laminar boundary layer according to expression (34) (curve 2) intersects function $\tilde{\Psi}_\delta$ for a turbulent layer only within a narrow range of relatively small δ^+. Curve 3 here corresponds to the outer segment of a typical Blasius profile, which constitutes a continuation of the integral in Newton's law at the wall

$$u^+ = y^+ \tag{35}$$

and is bounded by curve 2.

The graph in Fig. 3 is convenient in that it not only indicates the flow conditions (turbulent or laminar) depending on the value of δ^+ but also the profile of mean longitudinal velocity.

Expressions (14), (16), (18), (28), and (31)-(33) simplify the characteristics of molecular transfer. They become particularly convenient in the case of simultaneous effects of several physical factors on the molecular transfer process.

With respect to relation (31), the molecular transfer process will be characterized by different values of the critical function $\tilde{\Psi}_\delta$, which depend on the influencing factors. In

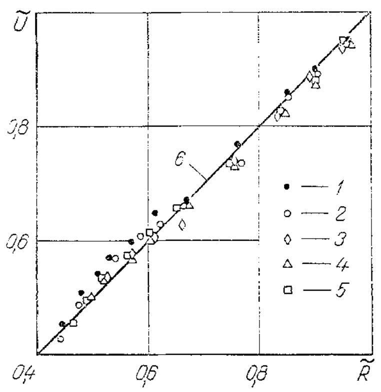

Fig. 1

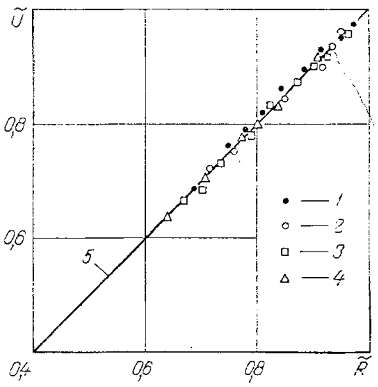

Fig. 2

Fig. 1. Distribution of generalized dimensionless velocity $\tilde{U}(\tilde{R})$ at Reynolds number $N_{Re} = 4.25 \cdot 10^4$ and various values of Hartmann number N_{Ha}: 1) $N_{Ha} = 0$, $\tilde{\Psi}_\delta = 0.31$, 2) 279 and 0.29; 3) 390 and 0.27; 4) 502 and 0.25; 5) 614 and 0.23; 6) according to relation (32).

Fig. 2. Distribution of generalized dimensionless velocity $\tilde{U}(\tilde{R})$ of turbulent flow in initial segment of rough pipe: 1) $x/d = 1.5$, $\tilde{\Psi}_\delta = 0.58$; 2) 3.5 and 0.46; 3) 9.5 and 0.44; 4) 25.5 and 0.44; 5) according to relation (32).

order to establish quantitative relations for molecular transfer occurring under complex conditions, it is necessary to know the relation $\tilde{\Psi}_\delta = \tilde{\Psi}_\delta(K_1, K_2, \ldots, K_n)$, where any of parameters K_i characterizes the dependence of molecular transfer on some influencing quantity such as, for instance, the magnetic induction in the case of an electrically conducting fluid.

Let $\tilde{\Psi}_{\delta 0}$ be defined by expression (33), i.e., $\delta^+ \to \infty$. As has been established earlier [1],

$$\tilde{\Psi}_{\delta 1}(K_1) = \tilde{\Psi}_{\delta 0} - \ln f(K_1), \tag{36}$$

$$\tilde{\Psi}_{\delta 2}(K_2) = \tilde{\Psi}_{\delta 0} - \ln f(K_2). \tag{37}$$

In the case of simultaneous dependence of the molecular transfer process on parameters K_1 and K_2 we have

$$\tilde{\Psi}_{\delta 12}(K_1, K_2) = \tilde{\Psi}_{\delta 0} - \ln f(K_1, K_2). \tag{38}$$

When

$$f(K_1, K_2) = f(K_1)\, f(K_2), \tag{39}$$

then

$$\tilde{\Psi}_{\delta 12} = \tilde{\Psi}_{\delta 0} - \ln f(K_1) - \ln f(K_2). \tag{40}$$

Condition (39) corresponds to a weak correlation between parameters K_1 and K_2.

It follows from relation (40) that, when condition (39) is satisfied, the relation $\tilde{\Psi}_\delta = \tilde{\Psi}_\delta(K_1, K_2, \ldots, K_n)$ can be determined according to the principle of superposition, which will greatly simplify calculation of the characteristics of molecular transfer under complex flow conditions. It is also important that, on the basis of relation (31), the principle of superposition can be applied to final characteristics ($\tilde{\Psi}_\delta$) as well as to instantaneous ones ($\tilde{\Psi}$).

As an example we will consider molecular transfer in rotating channels during acceleration of the stream due to change in their cross sections. On the basis of the model described here we obtain, analytically, the conditions of conservation of turbulent heat and mass transfer in such a system. During acceleration of the stream, according to experimental data [7],

$$\tilde{\Psi}_{\delta 0} - \tilde{\Psi}_\delta(K) = -C_1 K, \tag{41}$$

where the parameter $K = (\nu/u_\delta^2)(du_\delta/dx)$ characterizes the pressure gradient. Here $C_1 = 1.2 \cdot 10^4$. As the parameter K increases, the stream becomes laminarized with $\tilde{\Psi}_\delta(K) > \tilde{\Psi}_{\delta 0}$. When channel rotates at an angular velocity ω about the z axis, then, according to experimental data [8, 9],

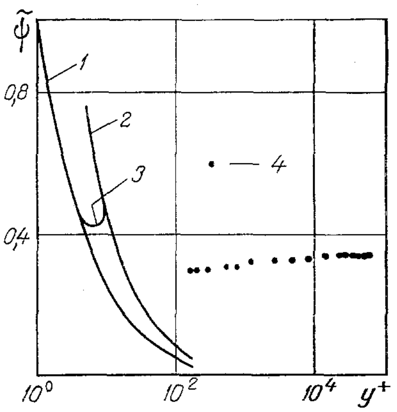

Fig. 3. Dependence of $\tilde{\Psi}$ on y^+: 1) according to expression (35); 2) according to expression (34); 3) typical Blasius profile; 4) values of $\tilde{\Psi}_\delta$ according to data in [5].

$$\tilde{\Psi}_{\delta 0} - \tilde{\Psi}_{\delta}(\omega) = -C_2 N_{Ro}, \tag{42}$$

with the rotation number $N_{Ro} = 2\omega\delta/u_\delta$ and $C_2 = 2/3$. As the angular velocity ω decreases ($\omega < 0$), the stream becomes laminarized with $\tilde{\Psi}_\delta(\omega) < \tilde{\Psi}_{\delta 0}$.

Parameters K and N_{Ro} are weakly correlated to each other. Therefore, it follows from relations (40)-(42) that, although a turbulent stream becomes laminarized as K increases (or ω decreases), it is possible to make it remain turbulent according to relation (31) by simultaneously affecting $\omega(\omega < 0)$ and $K(K > 0)$. This is important in rotating systems where the intensity of heat transfer must not decrease during laminarization. It is then necessary that the relation

$$K/N_{Ro} = C_2/C_1 \tag{43}$$

hold true.

The generalized relations which have been obtained here for molecular transfer of momentum, heat, and mass are universal and simplify calculation of its characteristics.

NOTATION

u, mean longitudinal velocity, m/sec; T, mean temperature, °K; ν, kinematic viscosity, m^2/sec; ρ, density, kg/m^3; τ, shearing stress, N/m^2; $u_* = \sqrt{\tau_w/\rho}$, dynamic velocity, m/sec; δ, thickness of the boundary layer, m; δ_0, thickness of the laminar sublayer, m; $y^+ = yu_*/\nu$, dimensionless space coordinate; $u^+ = u/u_*$, dimensionless velocity; $U = (u^+ - u_0^+)/(u_\delta^+ - u_0^+)$, generalized dimensionless velocity; N_{Re}, Reynolds number; N_{Ha}, Hartmann number; subscripts: *, stream parameters at $y^+ = 1$; δ, stream parameters at $y = \delta$; 0, stream parameters at $y = \delta_0$; and W, wall.

LITERATURE CITED

1. V. F. Potemkin, "Turbulence of boundary layer," Inzh.-Fiz. Zh., 39, No. 1, 39-46 (1980).
2. V. F. Potemkin, "Molecular transfer of momentum and heat," Inzh.-Fiz. Zh., 41, No. 3, 441-448 (1981).
3. L. G. Genin, V. G. Zhilin, and B. S. Petukhov, "Experimental study of turbulent flow of mercury in circular tube in longitudinal magnetic field," Teplofiz. Vys. Temp., 5, No. 2, 302-307 (1967).
4. Van Zhen-Sun and Tallis, "Turbulent flow in initial segment of rough pipe," Trans. ASME, Ser. D: Theoretical Principles of Engineering Calculations, No. 1, 158-165 (1974).
5. J. Nikuradse, "Laws of turbulent flow in smooth pipes," Contribution to Engineering Research, No. 356 (1932).
6. H. Blasius, "Boundary layers in fluids with low friction," Z. Angew. Math. Phys., 56, 1-37 (1908).
7. J. Nikuradse, "Studies of turbulent fluid flow in diverging and converging channels," Contribution to Engineering Research, No. 289 (1929).

8. Johnston, "Suppression of turbulence during shear flow in rotating systems," Trans. ASME, Ser. D: Theoretical Principles of Engineering Calculations, No. 2, 131-140 (1973).
9. Koyama, Masuda, Ariga, and Vatanabe, "Stabilizing and destabilizing effect of Coriolis force on two-dimensional laminar and turbulent boundary layers," Trans. ASME, Ser. B: Power Machinery and Apparatus, No. 1, 27-35 (1979).

DEVIATION OF THERMAL ANEMOMETER SENSORS WITH SAGGING WIRES FROM THE COSINE LAW

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The authors describe a calculation method and determine numerical values for the influence of the sag of the measuring wire of a thermal anemometer sensor on the deviation from a cosine law.

In determining the absolute magnitude of the velocity vector in three-dimensional flows using thermal anemometer sensor wires one measures the magntiude of the effective component of the flow velocity, which influences the heat tranfer between the wire and the flow. These quantities are related by the cosine law [1]:

$$V_\delta = V\cos\delta. \tag{1}$$

When one allows for the influence of the longitudinal velocity component on the heat transfer the cosine law takes the form [1]

$$V_\delta = V(\cos^2\delta + k^2\sin^2\delta)^{1/2}\,. \tag{2}$$

These relations are derived on the assumption that the measuring wire of the thermal anemometer sensor is straight. However, this condition does not hold in actual sensors. The deviation of the measuring wire from the straight condition stems from technical causes in the sensor manufacture, and also from the linear thermal expansion of the wire.

We now derive the relation between the magnitudes of the effective component and the flow velocity vector for the case of a sagging wire, when the wire forms the arc of a circle. The effective component of the velocity vector in the segment of arc QP of the measuring wire DQA (Fig. 1) varies from $V_{\delta 1}$ to $V_{\delta 2}$. The area of the figure FTQP is

$$S_{FTQP} = S_{OFT} - S_{OPQ}. \tag{3}$$

Using the notation

$$OP = OQ = r, \tag{4}$$

we obtain

$$PF = V_\delta t, \tag{5}$$

$$S_{FTQP} = \frac{1}{2}\int\limits_{\delta_1}^{\delta_2}(r+V_\delta t)^2\,d\delta - \frac{1}{2}\int\limits_{\delta_1}^{\delta_2} r^2 d\delta = \int\limits_{\delta_1}^{\delta_2}\left(rV_\delta t + \frac{1}{2}V_\delta^2 t^2\right)d\delta. \tag{6}$$

The average value of the integrand is determined by the relation [2]

$$\left(rV_\delta t + \frac{1}{2}V_\delta^2 t^2\right)_{\text{ср}} = \frac{180}{(\delta_2-\delta_1)\pi}\int\limits_{\delta_1}^{\delta_2}\left(rV_\delta t + \frac{1}{2}V_\delta^2 t^2\right)d\delta. \tag{7}$$

Denoting the right side of Eq. (7) by I, we finally obtain

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